Team Number:	APMCM1957

Problem Chosen:	B

2017 APMCM summary sheet

Glaze spraying is a key process in ceramic production, yet how to make the glaze coating uniform in various situations is a rather confusing issue. We have to set up different models at different times. If we can find a relatively universal strategy, much time can be saved.

In this paper, we adopt elliptic double β distribution model to obtain the thickness cumulative model on planes and solve Issue 1. Then we modify the basic model to make it applicable when dealing with curved surfaces and spheres. In this way, we deal with Issue 2 and 3 successfully.

As any free-form curved surface can be regarded as a superposition of several basic ruled curved surfaces, combinatorial use of piece-wise approximation and superstition can perfectly generalize our solution above, as shown in Issue 4.

We use MATLAB simulation to find the optimal solution. Sensitivity analysis of spray thickness and spray gun height is also made.

Key words: Spraying painting, Trajectory optimization, Path planning, coating thickness

Contents

1. Introduction

1.1 Problem Restatement

Background

- Glaze spraying is a process easy to realize automation in the ceramic production process. As uneven glaze will crack in the firing process and cause work piece scrapping, the thickness of the sprayed glaze in the spraying process is required to be as uniform as possible.
- In practice, the spray gun needs to move along the planned path and the mist cone will overlap in the adjacent paths, in order to ensure spray surface to be uniform.
- Spraying on different surfaces requires different trajectory planning and overlap interval. This is a common and key issue in the ceramic production industry.

The issues at hand

Issue 1: If the spraying direction of the spray gun keeps unchanged, calculate the cumulative situation of spraying in the plane and find out the suitable overlap interval of the spray gun trajectory.

Issue 2: For curved surface $z = -x^2 + x - xy$ $(-10 < x < 10, -10 < y < 10)$, determine whether the spraying interval calculated in issue 1 is applicable. If not, plan the trajectory again and calculate the new overlap interval, so that the glaze thickness difference is less than 10%.

Issue 3: If the spraying direction of the spray gun is always the normal direction of the spraying point of the mist cone center, and other conditions remain unchanged, recalculate the result of issue 2.

Issue 4: Determine whether the result of issue 3 is applicable for any curved surface. Find a general solution to the spray path planning.

1.2 General Assumptions and Notations

Assumptions

☆ No emergency incidents happen.

☆ Different spray surfaces are equally rough.

☆ Mainly consider the static model

☆ We do not take into consideration diameter of glaze particles.

☆ Only consider one overlap.

☆ Paint is continuously sprayed without interruption

Notations and Explanations

Table 1 Variable Description

Notations	Explanations
a	Semi-major axis
b	Semi-minor axis
$Z(x, y)$	Accumulation of thickness
Z_{max}	Maximum thickness of the paint film
$Z_i(x, y)$	The i-th spraying track thickness accumulation
Z_{avg}	The average thickness

2. Problem Analysis

The process of analysis for the four problems are as Figure 2.1- Figure 2.4.

2.1 Analysis of problem 1

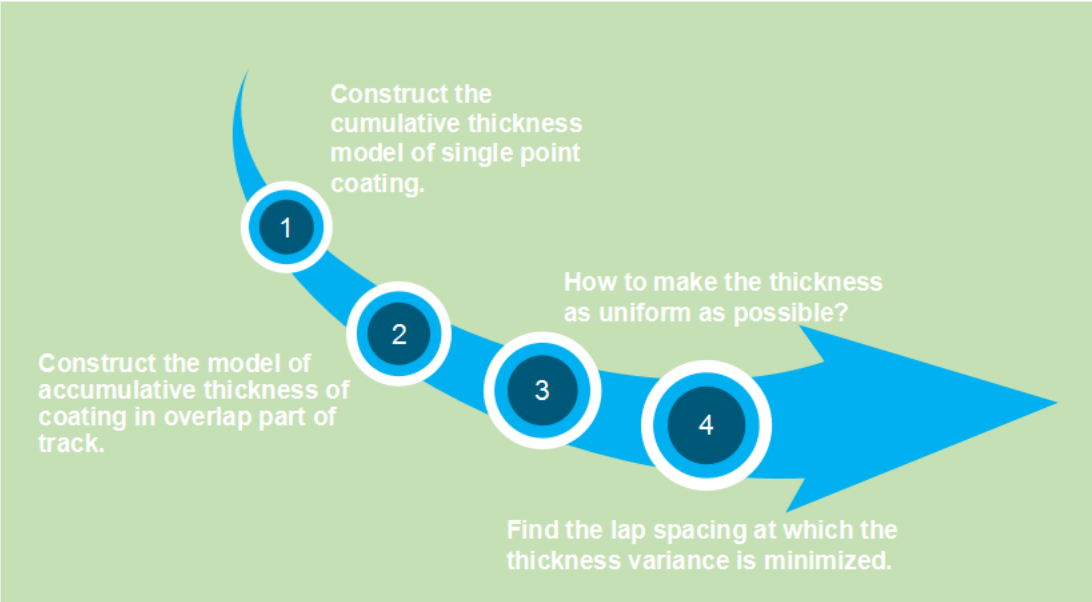

Figure 2.1 Flow chart of analysis of problem 1

2.2 Analysis of problem 2

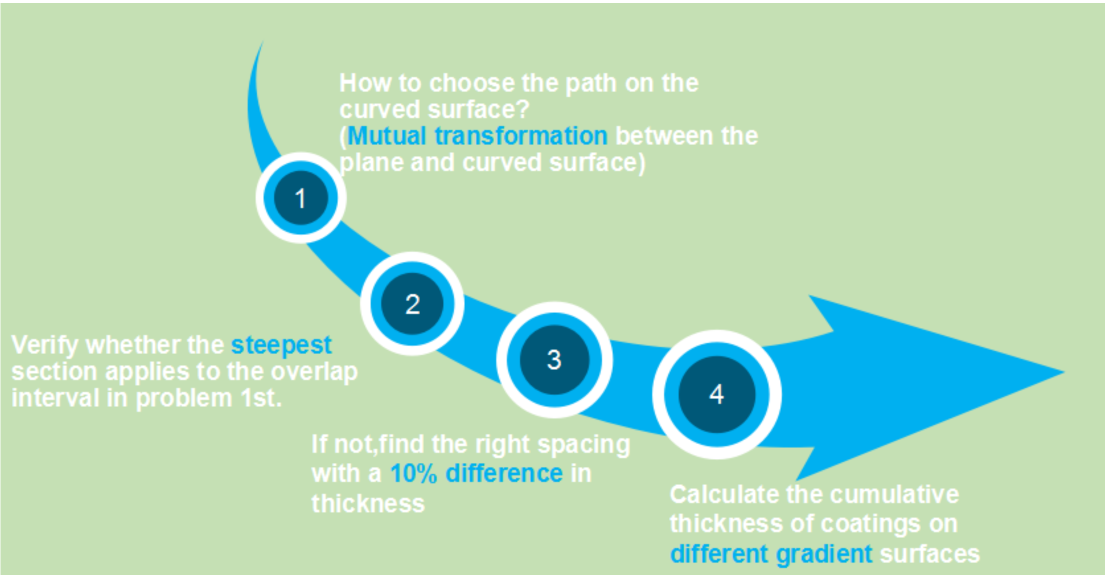

Figure 2.2 Flow chart of analysis of problem 2

2.3 Analysis of problem 3

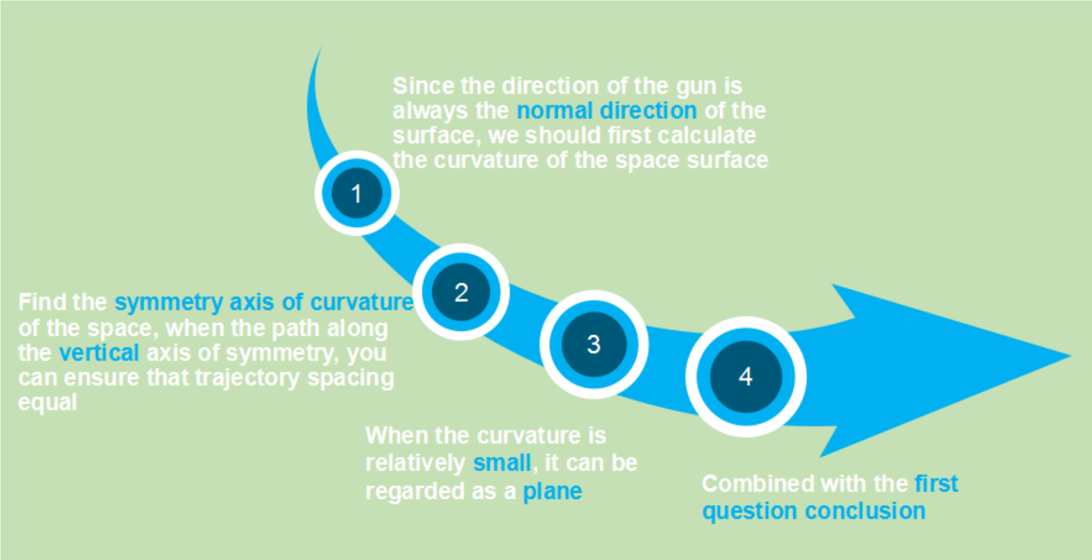

Figure 2.3 Flow chart of analysis of problem 3

2.4 Analysis of problem 4

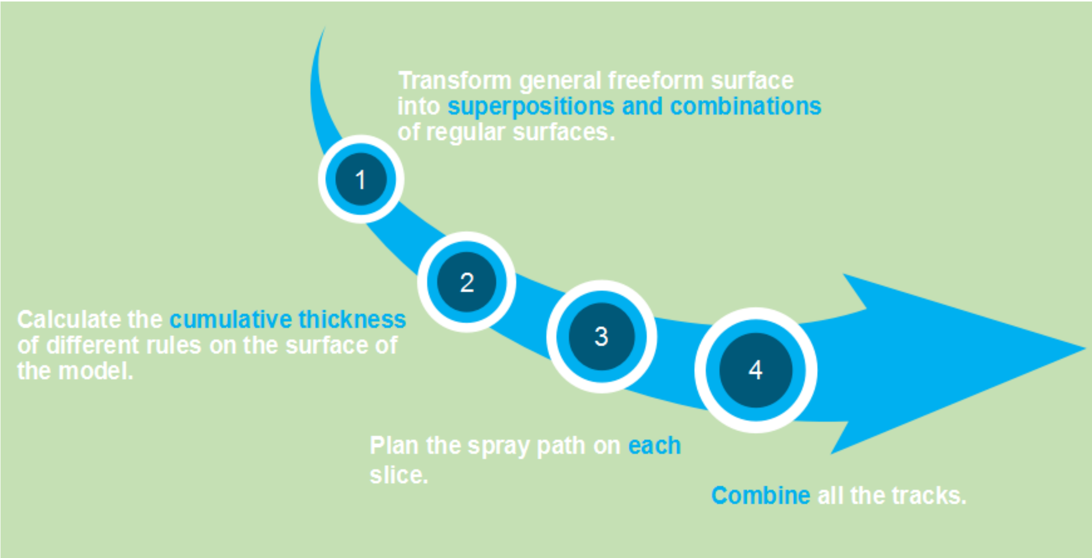

Figure 2.4 Flow chart of analysis of problem 3

3. Models and Justifications

3.1 The model of problem 1

3.1.1 Assumptions

- If the spray gun moves along the semi-minor axis of the ellipse, a larger area can be covered at a time and there are fewer repeated paths. Considering the issue of cost, we assume that the spray gun moves along the semi-minor axis.

- Assume that projection plane of XOZ isn't influenced by movement of the spray gun.

3.1.2 Establishment of model

- Considering the spraying region on the plane is an ellipse. It meets elliptic double β distribution model in the elliptic distribution region.

- After determining the parameters, we can simulate the model of thickness accumulation easily. As XOY section isn't affected by movement of the spray gun, according to our assumption, the model can be simplified in this issue.

- Then we take into consideration overlap interval of the spray gun trajectory and discuss the model in different parts of the plane.

- In order for thickness to be as uniform as possible, we take the interval that makes the variance the smallest.

3.1.3 Model solving process

We calculate the parameters with MATLAB and they are:

$$a=162.1518$$

$$b=69.2632$$

$$z_{max} = 209.0344$$

$$\beta_1 = 3.2175$$

$$\beta_2 = 5.2389$$

Model and simulation result is shown below.

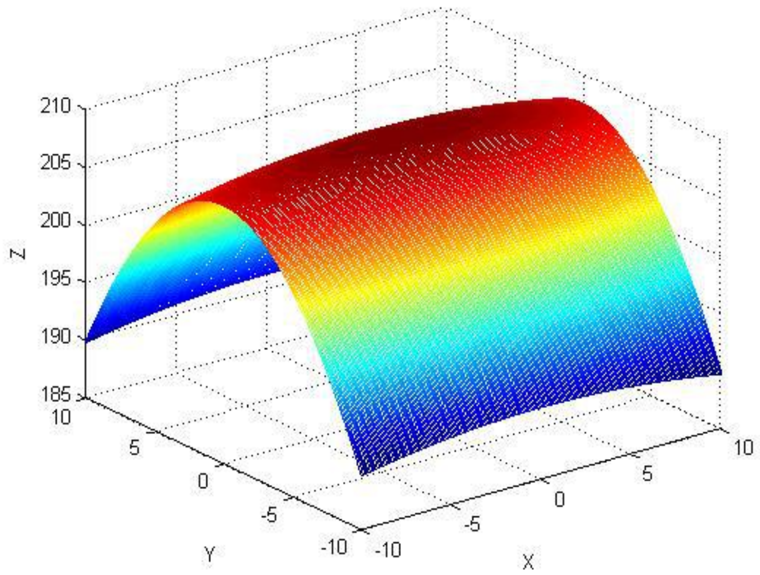

Figure 3.1 Diagram of elliptic double β distribution model (unit: mm)

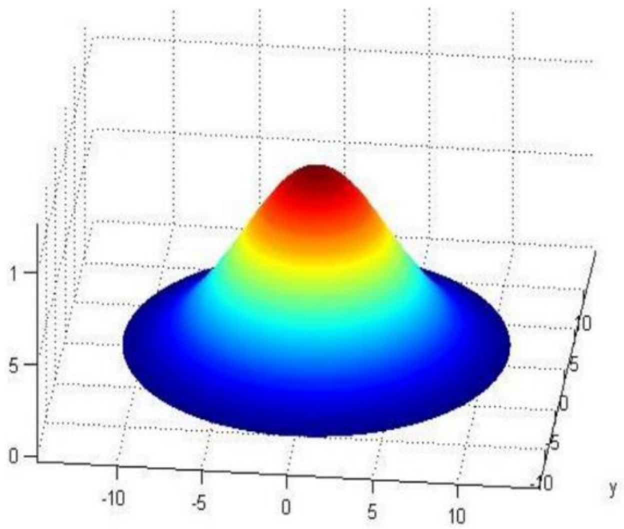

Figure 3.2 Diagram of elliptic double β single-point model (unit: mm)

When the gun moves in the direction of the minor axis(b), the spray area covers a greater area than along the major axis(a), so the spray gun is selected to spray along the minor axis of the oval spray area. [3] And projections of elliptic double β distribution model on XOZ and YOZ surface are as Figure 3.3 and Figure 3.4.

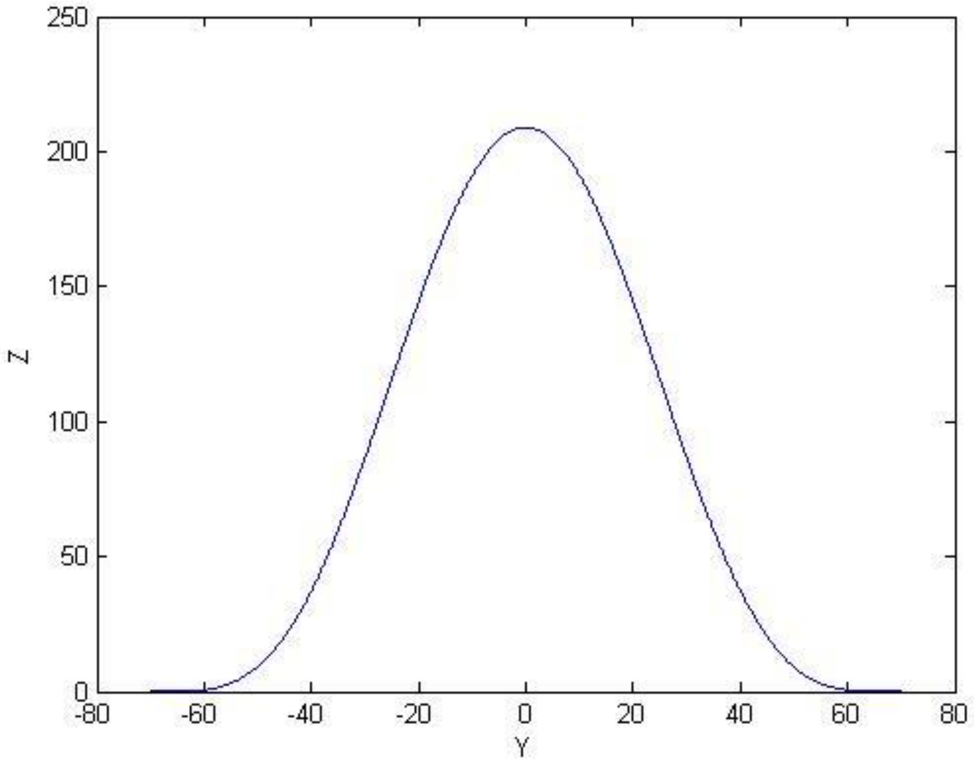

Figure 3.3 Projection of elliptic double β distribution model on ZOY

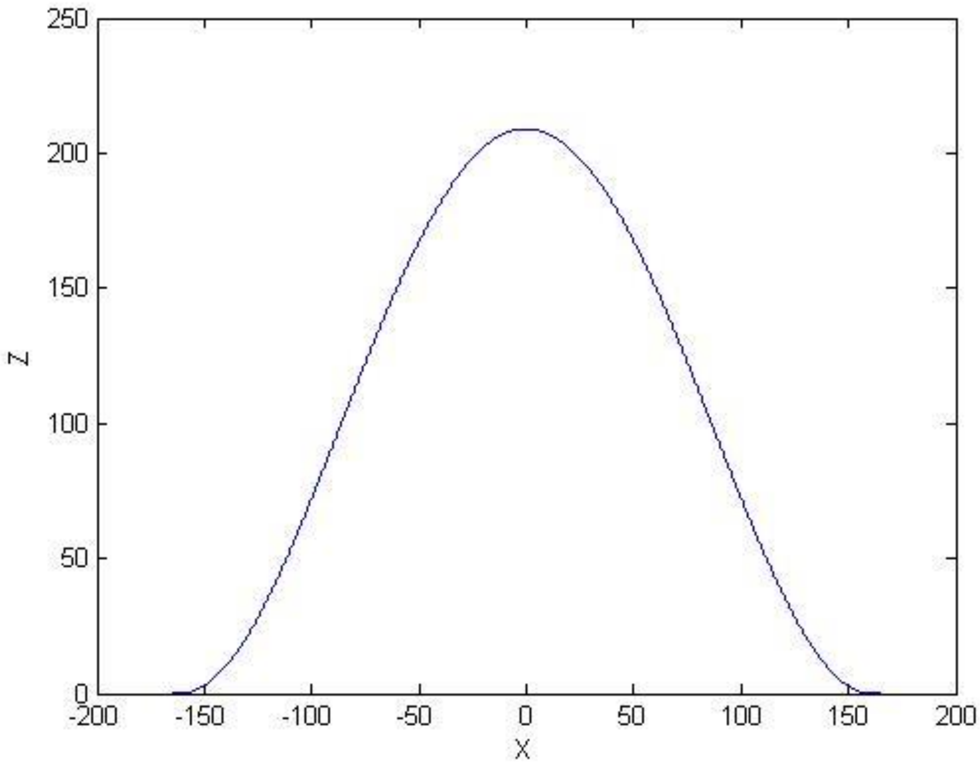

Figure 3.4 Projection of elliptic double β distribution model on XOZ

We analyze the overlap area of the adjacent paths. And suppose the overlap interval to be $x_0 (x_0 \in [a, 2a])$

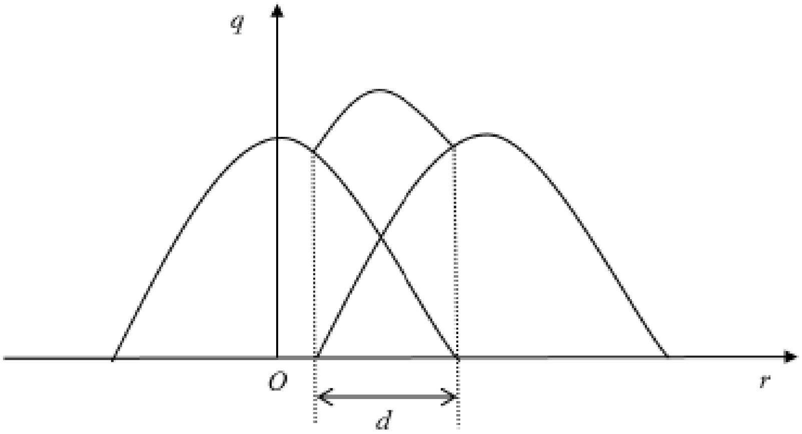

Figure 3.5 Schematic of overlapping

Then, we set Z_1 and Z_2 as two target paths, and we have:

$$Z_1(x, y) = Z_{max}\ (1 - \frac{x^2}{a^2})^{(\beta_1 - 1)}$$

$$Z_2(x, y) = Z_{max}\ (1 - \frac{(x - x_0)^2}{a^2})^{(\beta_1 - 1)}$$

$$Z_{overlapping}(x, y) = Z_1(x, y) + Z_2(x, y)$$

Now we consider distribution situation of the overlap area where x is in $[0, x_0]$. Then we calculate the average thickness and the variance as follows:

$$H(x_0) = \int_0^{x_0} z(x)dx = 2\int_0^{x_0/2} z(x)dx = \frac{S}{x_0}$$

$$D(x0) = \int_0^{x_0} (z - H)^2 dx = 2\int_0^{x_0/2} (z - H)^2 dx$$

S is the integral result of z to x-axis when x in [-a, a], which is equal to 34814.

$$Z = Z_{max}(\left(1 - \frac{x^2}{a^2}\right)^{B1-1} + \left(1 - \frac{(x - x_0)^2}{a^2}\right)^{B1-1}), \quad x_0 - a \le x < x_0/2$$

$$Z = Z_{max}\left(1 - \frac{x^2}{a^2}\right)^{B1-1}, \quad 0 \le x < x_0 - a$$

So the issue change into that finding the x_0 when $D(x_0)$ reach the minimum value. We define $d(x_0)$ as $D'(x_0)$ and draw the figure of $d(x0)$ with MATLAB, as Figure 3.6.

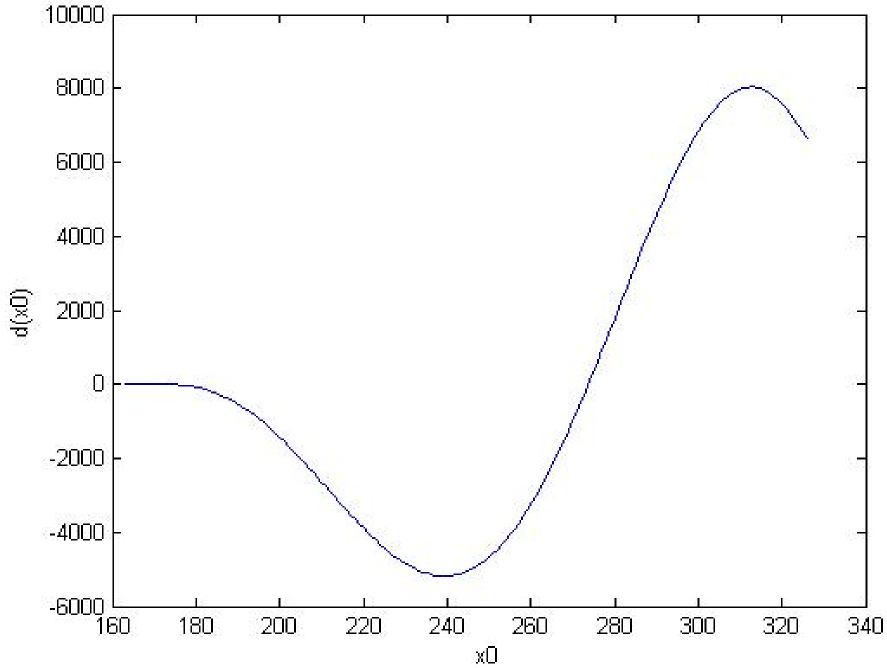

Figure 3.6 Function graph of $\mathrm{d}(x_0)$ (unit:mm)

From Figure 3.6, we can know that when $x_0 = 274$, $\mathrm{d}(x_0) = 0$.

And $x_0 \in [\mathrm{a}, 274]$, $\mathrm{d}(x_0) < 0$; $x_0 \in [274, 2\mathrm{a}]$, $\mathrm{d}(x_0) > 0$, so $x_0 = 274$, $\mathrm{D}(x_0)_{min}$. When $\mathrm{d}(x_0) = 0$, the x_0 is the most suitable overlap interval for adjacent spray track.[6]

Thus, we can come to the conclusion that **274mm** is the best overlap interval under the condition of problem 1.

3.2 The model of problem 2

Firstly, we draw the figure of curved surface with MATLAB. The equation is $z = -x^2 + x - xy(-10 \le x \le 10, -10 \le y \le 10)$, showed as follow.

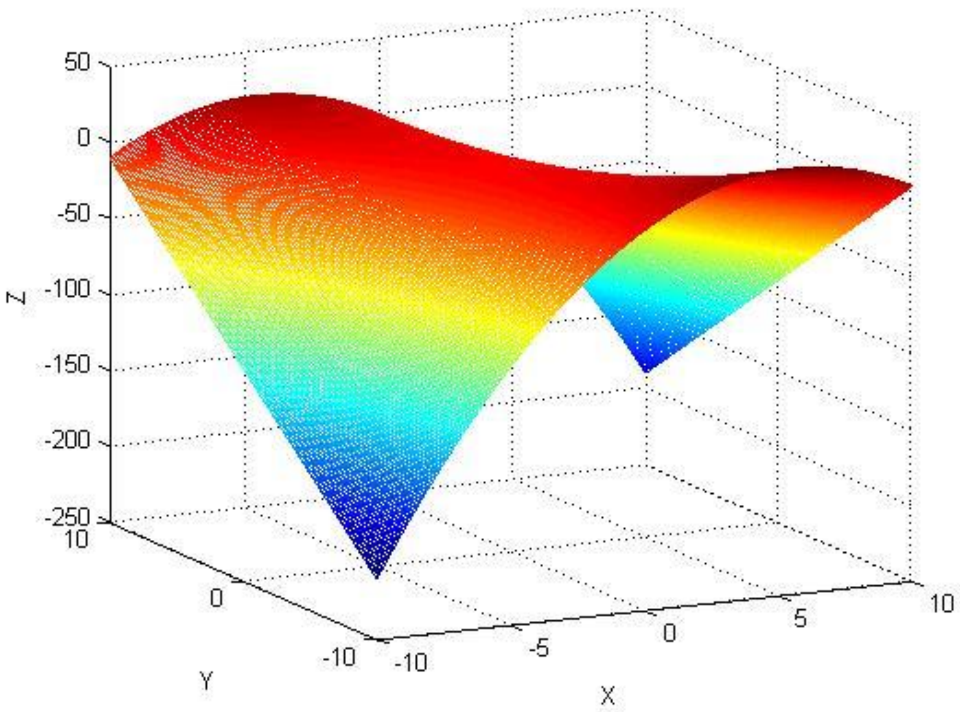

Figure 3.7 3D figure of the curved surface (unit: m)

In this case, the spraying direction of the spray gun remains unchanged, which is vertical. However, the surface of the sprayed item is changed into a curved surface from flat. The schematic of spraying is showed as Figure 3.8.

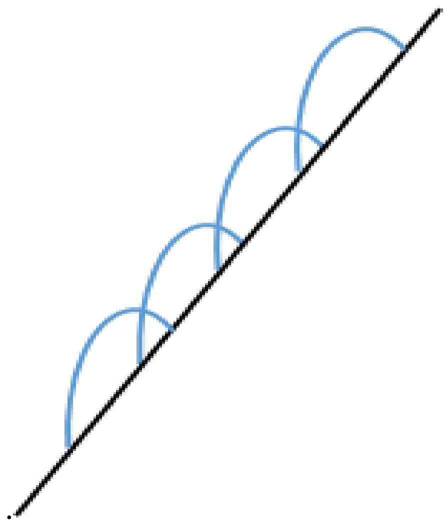

Figure 3.8 Working schematic of spray gun

3.2.1 Assumptions

The normal direction is perpendicular to the horizontal plane.

3.2.2 Spray gun trajectory

In order to discover surface features, we also draw the contour map and gradient vectors of the curved surface, showed as Figure 3.9.

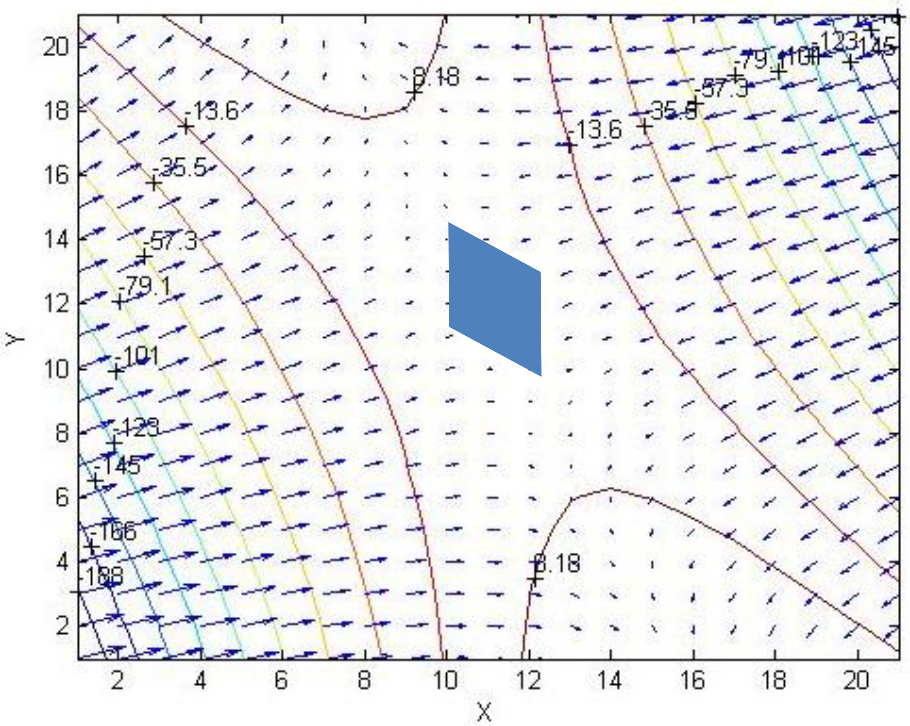

Figure 3.9 contour map of the curved surface (unit: m)

The literature[1] points out spray path should be equally divided item to make spray surface to uniform.

And from Figure 3.9, we can see the contour of curved surface is symmetrical. So we determine the spray gun path is the direction of the contour.

The direction also makes sure maximum height of some spray gun path is equal.

In the area of sparse in the contour, we can see curved surface as flat, so we can use the spraying interval in those area, such blue area in Figure 3.9

3.2.3 Solution and Result

If the spraying interval calculated in issue 1 is applicable?

If we want to determine the result of issue 1 is applicable, we need calculate minimum height in the maximum gradient area and judge if the difference between minimum height and maximum height is less than 10%.

We calculate the gradient of z as follow:

$$grad(x,y) = (-2x - y + 1)\ e_x + (-x)e_y$$

When $x = -10$ and $y = -10$, $|grad(x,y)_{max}| = 32.5730$, and we have:

$$gard(-10,-10) = (31)e_x + (10)e_y$$

We can set

$$z = 32.5730x.$$

When $x = 274mm = 0.274m$, $z = 8.925m$. It means that when the spray gun is translated 274mm, the center point of spray gun increasing by 8.925m. Figure 3.10 shows the situation.

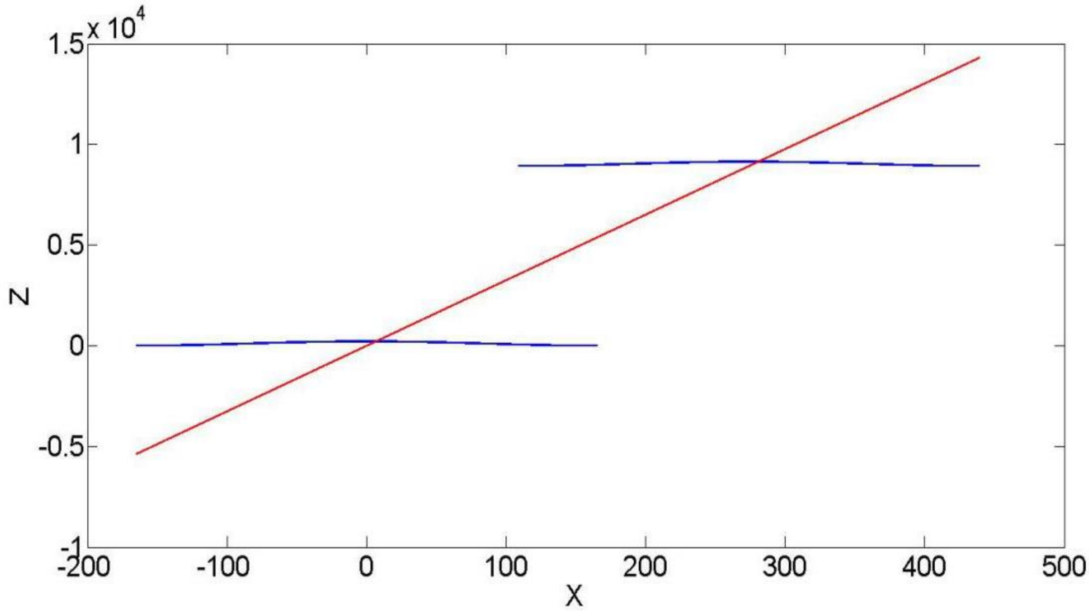

Figure 3.10 Working schematic of spray gun (unit: mm)

In this case, the overlapping area is greatly different from the overlap area in the plane, so we adopt the thickness model on the curved surface.

Curved Surface Thickness Model(CSTM)[1]

Spray gun spray on the surface of the spray as shown in **Figure 3.10.**

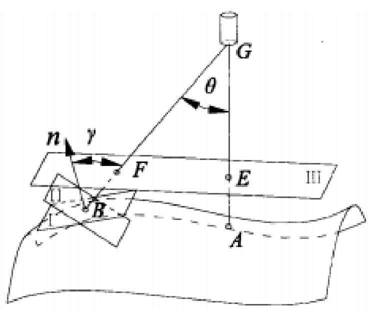

Figure 3.10 Schematic of CSTM

The coating thickness model on the surface is proposed by Chen, expressed as:

$$q_B{*} = q_B \cos\gamma = q_F\left(\frac{h}{l}\right)^2 \frac{\cos\gamma}{\cos^3\theta}$$

Table 2 Notations of CSTM

Notations	Explanations
$q_B{}^*$	coating thickness of a point B on the surface mapping plane I
q_B	coating thickness of a point B on the surface mapping plane II
q_F	coating thickness of a point B on the surface mapping plane III
γ	the angle between the normal vector **n** at the point B and line GB
θ	spray angle

Table 2 shows some notations of CSTM. And,

$$|GE| = h$$

$$|GB| = l$$

In our issue,

$$tan(\theta) = 32.5730$$

So,

$$cos(\theta) \approx 1$$

And normal vector of the curved surface is

$$\boldsymbol{n} = (-2x - y + 1, -x, -1)$$

$\boldsymbol{n} = (31, 10, 1)$ at point B$(-10, -10, -210)$

And we set

$$h = 225mm$$

G point is the result of B point translation 274mm along the gradient direction of B point while G is 225mm higher than B. So we get the coordinate of G point as:

$$G\left(-10 + \frac{0.274}{32.5730} \times 31, -10 + \frac{0.274}{32.5730} \times 10, -210 + 8.925 + h\right)$$

The result is,

$$G(-9.74, -9.91, -200.85)$$

And we can get

$$cos(\gamma) = \frac{|\overrightarrow{GB} \bullet \vec{\boldsymbol{n}}|}{|\overrightarrow{GB}| \bullet |\vec{\boldsymbol{n}}|} = 0.09$$

$$l = \left|\overrightarrow{GB}\right| = 33.83$$

$$\mathrm{q}_f \approx \mathrm{z}_{max} = 0.209$$

So,

$$q^{*} \approx 1.067 \times 10^{-6}$$

Result

There is almost no effect on the (-10, -10, -210) points after the spray gun is translated by 276 mm, which means the thicknesses **hardly increase**. Thus, **spraying interval calculated in issue 1 isn't applicable for issue 2.**

Calculate the overlap interval again

In order to ensure that the thickness difference is 10%, it is necessary to ensure that the track interval is sufficient. And in this case we regard z_{max} as z_{avg}.

The problem turns to solving the equation:

$$Z_{max}\left(1-\frac{x^{2}}{a^{2}}\right)^{B1-1} = 0.9Z_{max}$$

We set h=255mm, and get x=34.92mm, hence come to the result that

$$spraying\ interval \in [\mathbf{34.92}\boldsymbol{mm}, \mathbf{276}\boldsymbol{mm}]$$

The value of spraying interval depends on the **gradient of the surface.**

3.3 The model of problem 3

For complex surfaces, we use a sub-path planning of the seed curve to segment each run to optimize the velocity for each run so that the coating builds up along the gun travel direction and then optimizes the spacing between adjacent runs , So that the coating thickness reaches the desired film thickness at any point between the runs. This method avoids re-modeling the rate of coating build-up. The cumulative rate of coating on the surface can be calculated from the rate of coating build-up on the surface, and the uniformity of the coating can be improved by increasing the number of segments per pass. And This method reduces the difference between the actual coating thickness and the desired film thickness.[8]

Here we first calculate the curvature of the surface, using the sum of the curvature of the surface z in the x direction and y direction as the square of curvature, showed as Figure 3.11. And Figure 3.12 shows the project the curvature on the XOY plane.

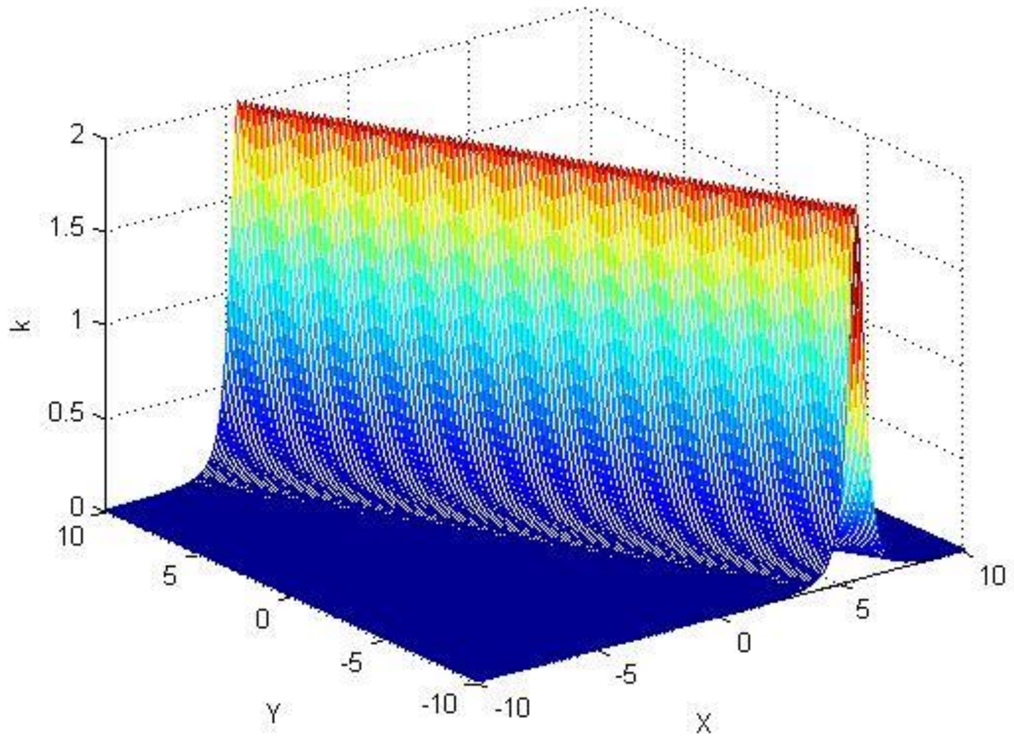

Figure 3.11 3D figure of square of curvature of the curved surface (unit: m)

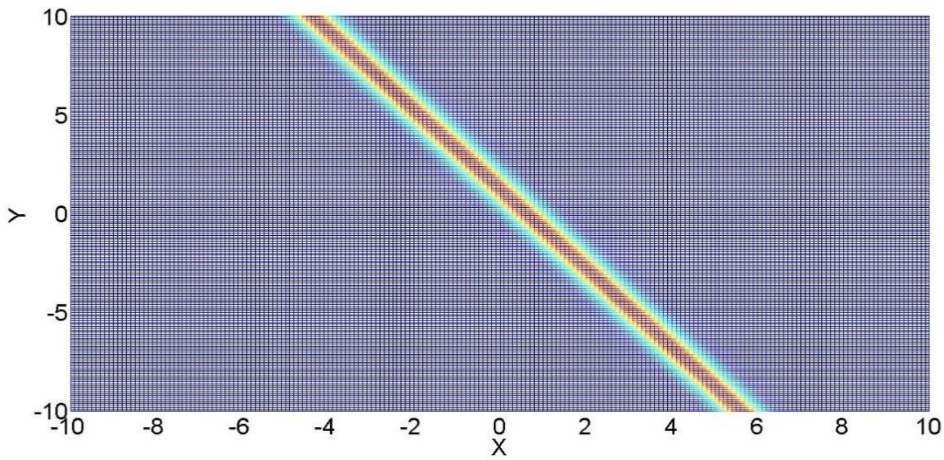

Figure 3.12 Project the curvature on the XOY plane (unit: m)

In this case, the spraying direction of the spray gun is always the normal direction of the spraying point of the mist cone center during spraying. And height of the path center is invariant.

Thus, in the surface with small curvature, the curved surface can be approximated as a plane in the space rotated by a certain angle, the result of problem 1 can be used at this time. In the surface with larger curvature, the overlap area between the two paths decreases, so the interval needs to be recalculated.

The schematic of spraying is showed as Figure 3.13.

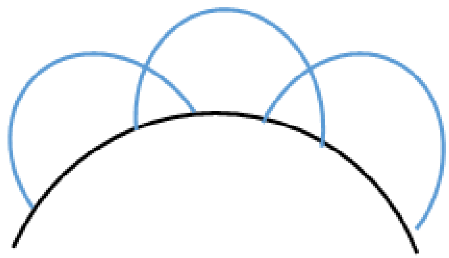

Figure 3.13 The schematic of spraying

3.3.1 Spray gun trajectory

Projection of the curvature in the XOY plane is symmetric about line $y = -2x + 1$. As a whole, in order to reduce the geodesic curvatures of the left and the right boundary paths, the seed curve must be put in the proper place, so the trajectory divides the surface into two parts with equal integral of curvature. [8] Curvatures of different paths change in the same way and interval between paths is a constant.

The projection of the spray trajectory on XOY plane should be the direction of the normal vector of line $y = -2x + 1$, which is (2, 1) , as shown in Figure 3.14.

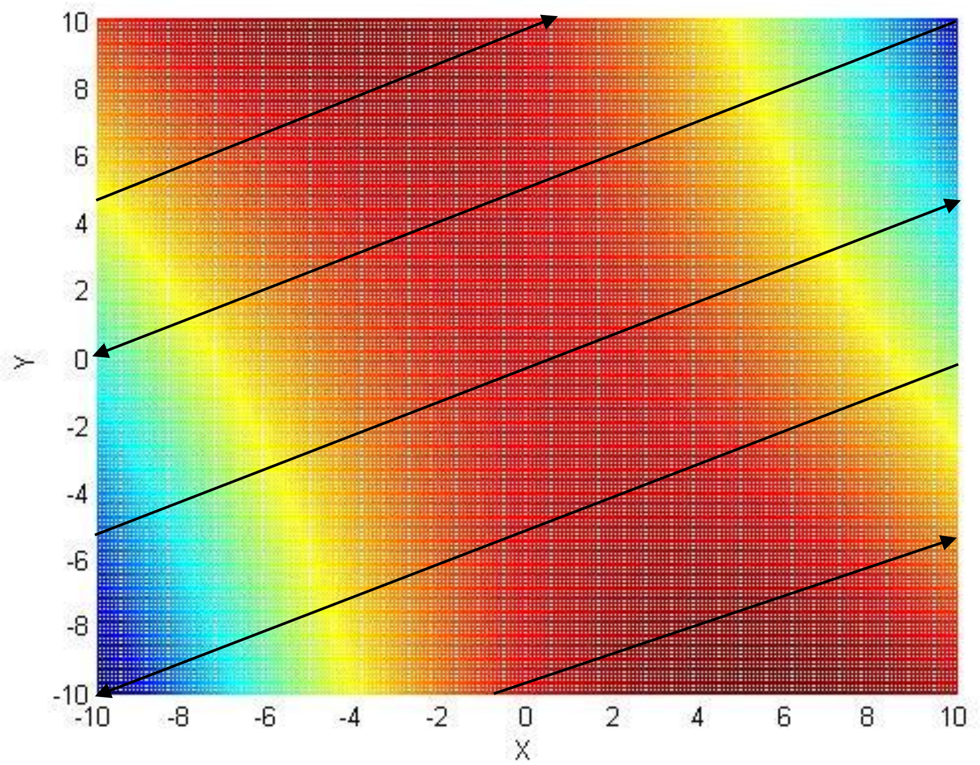

Figure 3.14 Paths of spray gun

3.3.2 Calculate spraying interval

For the parts whose curvature is large we adopt static spray model on spheres[9] rather than that on planes.

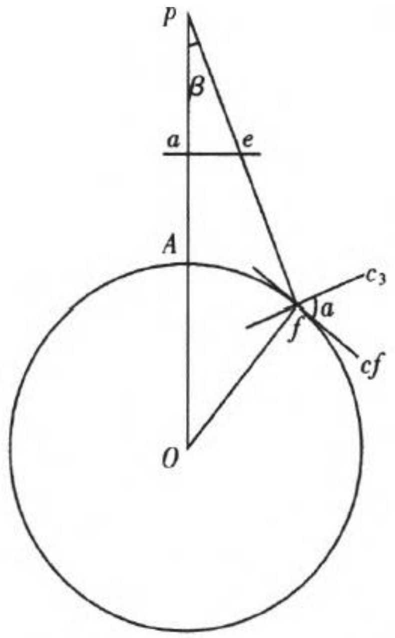

Figure 3.15 Plane Projection of Spraying on Spherical Surface

Parameters in Figure 3.15 satisfy the following relationship:

$$pA = h_A$$

$$pf = l$$

$$\cos\alpha = \frac{(h_A + R)^2 - l^2 - R^2}{2lR}$$

$$\cos\beta = \frac{(h_A + R)^2 + l^2 - R^2}{2l(h_1 + R)}$$

$$q_f = \begin{cases} q_e \dfrac{4h^2 (h_A + R)^3 [(h_A + R)^2 - l^2 - R^2]}{R[(h_A + R)^2 + l^2 - R^2]^3} & \alpha < 90° \\ 0 \quad \alpha \geq 90° \end{cases}$$

To ensure the least spraying times, we need to set the trajectory interval to be as big as possible. So we take the biggest deviation(10%) as our calculating condition.

That's to say,

$$|z_{max} - z_{min}| = 0.1 z_{max}$$

As a result,

$$z_{min} = 0.9 z_{max}$$

When $q_f = 0$, Z is the smallest, as the red dots in the following figure show.

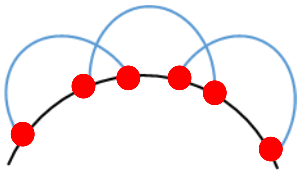

Figure 3.16 The schematic of spraying

In the Figure 3.17, $q_f(\text{red}) = 0$ when the red dot is in the red area and $q_f(blue) = 0.9q_A$ when it's in the blue area.

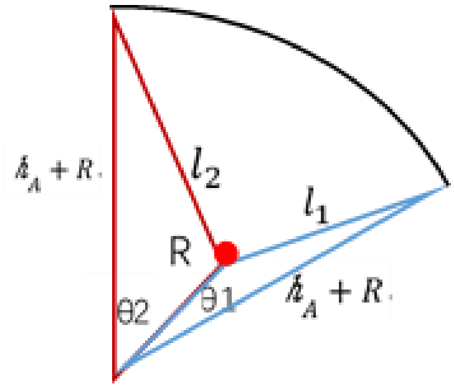

Figure 3.17 Schematic of overlapping on sphere

For the blue triangle:

$$q_A = z_{max}$$

$$z_{max} = 240.41 - 0.1244h_A$$

$$q_f = 0.9q_A$$

$$q_f = \begin{cases} q_e \dfrac{4h^2(h_A+R)^3[(h_A+R)^2 - l^2 - R^2]}{R[(h_A+R)^2 + l^2 - R^2]^3} & \alpha < 90° \\ 0 \quad \alpha \geq 90° & \end{cases}$$

$$q_e = z_{max}\left(1 - \frac{|ae|^2}{a^2}\right)^{\beta_1 - 1}$$

$$\mathrm{a} = 1.7436h_A - 282.45$$

$$\beta_1 = 0.0284h_A - 4.0389$$

$$|ae| = |\mathrm{pa}|\tan\beta$$

$$\cos\beta = \frac{(R + h_A)^2 + {l_1}^2 - R^2}{2l_1(R + h_A)}$$

Then we can obtain a function of l_1 about h_A and $|pa|$, and $|pa| < h_A$.

For the red triangle, if $q_f = 0$, then

$$l_2 = \sqrt{{h_A}^2 + 2h_A R}$$

So we can obtain a function of l_2 about h_A.

According to cosine theorem:

$$\cos(\theta_1) = \frac{R^2 + (R + h_A)^2 - {l_1}^2}{2R(R + h_A)}$$

$$\cos(\theta_2) = \frac{R^2 + (R + h_A)^2 - {l_2}^2}{2R(R + h_A)}$$

$$\theta = \theta_1 + \theta_2$$

And we have,

$$R = 500mm$$

So the maximum interval distance is $\theta(R + h_A)$ (Black arc in Figure 17)

3.2.3 Solution and Result

We set

$$h_A = 255mm$$
$$|pa| = 200mm$$

We can get

$$maximum\ spraying\ interval = 692.1085mm \text{ (for maximum curvature)}$$

The value of spraying interval：

$$spraying\ interval \in [276mm, 692.1085mm]$$

The value of spraying interval depends on the curvature of the surface.

But we want to make the overlapping interval constant. And the maximum spraying interval decreases with curvature decreasing in the same path, so we need to satisfy the maximum spraying interval in the same path with the minimum curvature position.

At this point, we can set spraying interval in the plane as spraying interval in the whole curved surface, so we have,

$$spraying\ interval = \mathbf{276mm}$$

3.4 The model of problem 4

3.4.1 Spray Accumulation Models

Gaussian Distribution Model corresponds to the glaze distribution in practice better than other models, so the material adopt elliptic double β distribution to describe the accumulation state of glaze during the spraying process.The mist is squashed into an elliptical cone and accumulation of spray is :

$$z(x,y) = z_{\max}\left(1-\frac{x^2}{a^2}\right)^{\beta_1-1}\left[1-\frac{y^2}{b^2\left(1-\frac{x^2}{a^2}\right)}\right]^{\beta_2-1}$$

This model provides indexes β_1 and β_2 that can influence shape of the glaze-coat, so it can be used in very different situations.

Generally speaking, a free-form curved surface can be regarded as a superposition of several basic ruled curved surfaces (similar to adding several sinusoidal signals to form an actual signal spectrum in signal processing). Then we deal with the basic ruled curved surfaces one by one.

We list the following situations:

Plane

Horizontal planes can be directly handled with elliptic double β distribution model and thickness accumulation rate subjects to Gaussian distribution.

If the plane has a slope angle, effect of spraying, the time consumed and total length of the path can all be optimized in experimental simulation when the normal direction of the spray gun is perpendicular to the plane. In fact, this is a widely-used method of spraying in many industries. Therefore, even with a plane that has a slope angle, elliptic double β distribution model is still applicable.

Sphere

According to literature[2], thickness cumulative rate distribution during static spraying of spherical surface is showed as Figure 3.15.

Cylindrical surface

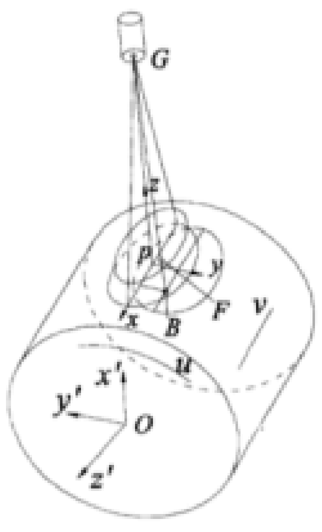

Figure 4.1 The moving frame on a cylindrical surface

Data in the literature[1] shows that thickness of the glaze increases with the radius. As a result, there will be less waste. It also indicates that the spraying effect is relatively better if the curvature of the curved surface is smaller.

Conical Surface

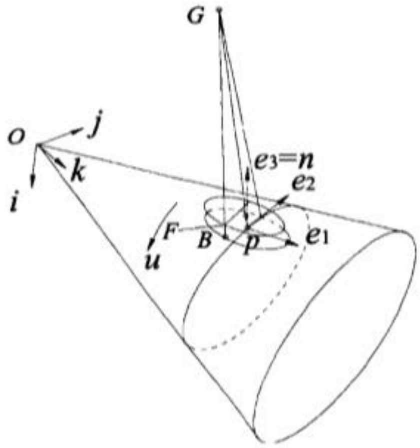

Figure 4.2 The moving frame model

Other Surfaces of Revolution

Intervals between lines of longitude on revolution surfaces are uneven at many times, while intervals between lines of latitude along the direction of longitudes are often even. Therefore we choose the lines of latitude as the spray trajectory.

Cumulative spray model on rotating paraboloids and schematic diagram of spraying trajectory are shown below:

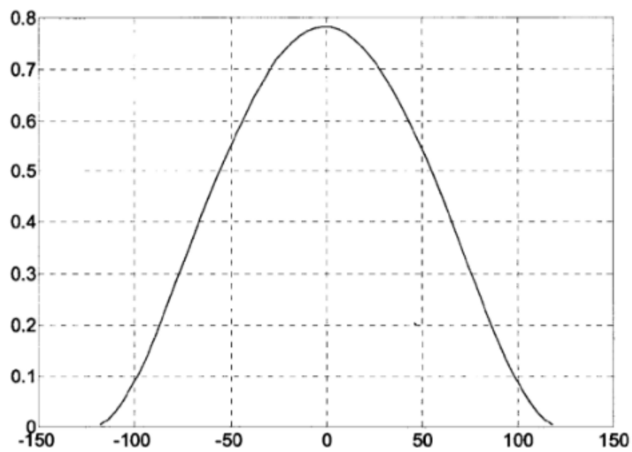

Figure 4.3 Rotating parabolic surface coating thickness cumulative model

（The vertical axis is spray thickness;

The horizontal axis is the long axis of the spray area;

Unit: mm）

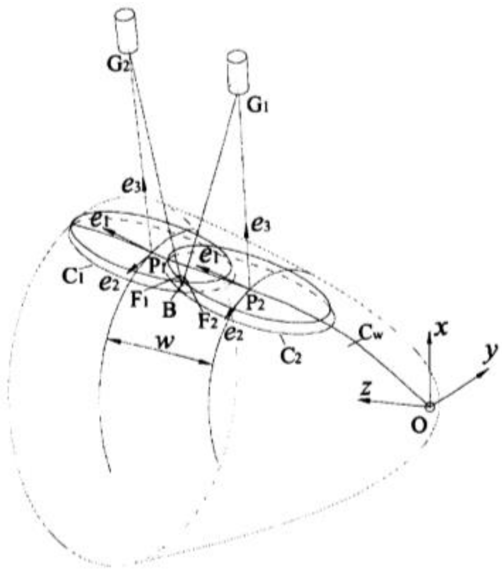

Figure 4.4 The moving frame model

3.4.2 A Brief description of spray trajectory planning

Plane

Solution in Issue 1 is rather typical. In order for the thickness of the sprayed glaze to be as uniform as possible, we choose the overlap interval that minimizes the variance of the thickness to obtain the trajectory.

Sphere

According to the principle of seed curve generation in literature[8], we take the lines of longitude as the spray trajectory by obtaining the set of geodesics. With the increase of geodesic curvature, the possibility of intersecting with other trips gradually increases. In order to reduce the possibility of self-intersection of the paths, the seed curve divides the surface into two parts with the same Gaussian curvature.

Cylindrical surface

The deviation of thickness along the arc path is less than that along the straight line. We should take into account both the deviation of thickness and the spray time. If the actual deviation meets the given demand, we can choose the shortest trajectory which consumes the least time. [1]

Conical surface

Judging from the shape of the cone and its parametric equation, we can see that the intervals are even if we spray along the curves parallel to the alignment while the spray gun moves at the same height and speed. If we spray along the generatric, the spray gun has to change its height and speed and the spray effect isn’t ideal. So we choose the curves parallel to the alignment as the spray trajectory. [1]

Other Surfaces of Revolution

Using lines of latitude as the spray trajectory is applicable in these issues.

Superposition

After obtaining the optimized trajectories of the basic ruled curved surfaces, we adopt the PA-PA (parallel-parallel) path to complete the synthesis of the optimized trajectory of the entire surface.[3]

4. Conclusions

4.1 Methods used in our models

In the process of building the model, we flexibly use the idea of calculus to segment the sprayed surface and simulate the spraying results through computer simulation. And the ideal experimental results are obtained in practice.

4.2 Advantages of our models

In response to the first question, we simplify the problem to find the cumulative part of the thickness of the cumulative model and variance.

According to the second problem, we divide the complex surface according to the curvature, treat it as a few planes, and make the best of the assumptions and conclusions in the first question.

In view of the general situation in the actual process, once again we implement the idea of slicing and transform the free-form surface into a combination of ordinary regular surfaces, which greatly simplifies the problem.

We use genetic algorithm and particle swarm optimization algorithm to optimize spray parameters, in order to get the most reasonable spray height.

4.3 Disadvantages of our models

We assume that the gun movement is uniform and does not take into account the actual production time and material costs, the gun should actually be variable speed movement.

We did not consider whether the paint coating is continuous and uniform under the actual situation and the influence of the paint particle size on the spraying effect.

5. Future Work

Triangulation of triangulation topography

According to the literature[7], Triangulation of triangulation topography can make the planning of spray paths more accurate and efficient.

We think that any free-form surface can be superposed or combined by ordinary rules and surfaces, which needs to be tested in practice.

Sensitivity Analysis of h

In those issues, we set h as 255mm. In fact, different values of h affect the shape of function Z, showed as Figure 5.1.

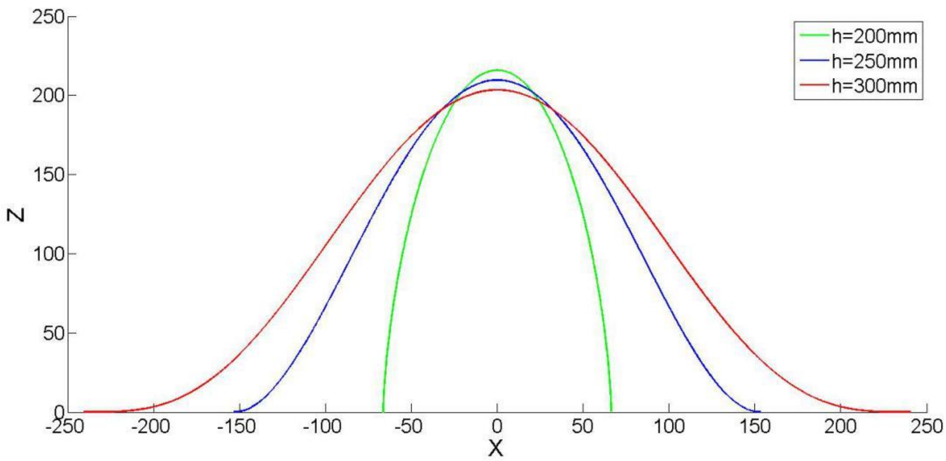

Figure 5.1 Shape of function Z change

When h=162.1mm, a=0, $z_{max}(max) = 220.59mm.$

When h=1935.3mm, $a(max) = 3091.7mm$, $z_{max} = 0mm.$

So,

$$h \in [162.1mm, 1935.3mm]$$

Thus, we can use genetic algorithms[9], ant colony algorithms[10] to optimize the value of h.

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Appendix

MATLAB code:

1. Calculate parameter

```
P1=0.2; %unit :Mpa
P2=0.2; %unit:Mpa
h=255; %unit:mm
A=[129.8665 -55.2435 1.7436 -297.3908;
    52.5130 -5.7480 0.7394 -128.6368;
    59.7245 393.9655 -0.1244 150.0184;
    -7.0125 34.5045 0.0284 -9.5229;
    -4.6130 18.3620 0.0113 -0.3924];
x=[P1;P2;h;1];
y=A*x
a=y(1,1);
b=y(2,1);
Zmax=y(3,1);
B1= y(4,1);
B2=y(5,1);
```

2. Draw figures

```
x=[-10:0.1:10];
y=[-10:0.1:10];
[X,Y]=meshgrid(x,y);
Z=-X.^2+X-X.*Y;
mesh(X,Y,Z)
xlabel('X')
ylabel('Y')
zlabel('Z')

figure
surf(X,Y,Z)
xlabel('X')
ylabel('Y')
zlabel('Z')

figure
h=contour(Z, 100);
clabel(h);
hold on;
```